

2-4

Solving Equations with Variables on Both Sides

Objective

Solve equations in one variable that contain variable terms on both sides.

Vocabulary

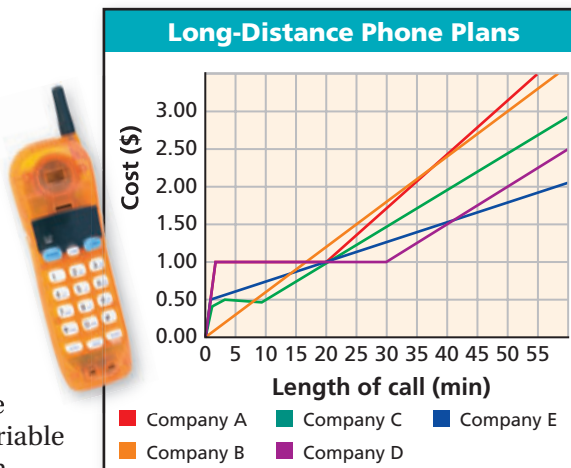
identity
contradiction

Why learn this?

You can compare prices and find the best value.

Many phone companies offer low rates for long-distance calls without requiring customers to sign up for their services. To compare rates, solve an equation with variables on both sides.

To solve an equation like this, use inverse operations to “collect” variable terms on one side of the equation.



EXAMPLE 1 Solving Equations with Variables on Both Sides

Solve each equation.

$$\begin{aligned} \text{A } 7k &= 4k + 15 \\ 7k &= 4k + 15 \\ \underline{-4k \quad -4k} & \\ 3k &= 15 \\ \frac{3k}{3} &= \frac{15}{3} \\ k &= 5 \end{aligned}$$

To collect the variable terms on one side, subtract $4k$ from both sides.

Since k is multiplied by 3, divide both sides by 3 to undo the multiplication.

$$\begin{aligned} \text{B } 5x - 2 &= 3x + 4 \\ 5x - 2 &= 3x + 4 \\ \underline{-3x \quad -3x} & \\ 2x - 2 &= 4 \\ \underline{+2 \quad +2} & \\ 2x &= 6 \\ \frac{2x}{2} &= \frac{6}{2} \\ x &= 3 \end{aligned}$$

To collect the variable terms on one side, subtract $3x$ from both sides.

Since 2 is subtracted from $2x$, add 2 to both sides to undo the subtraction.

Since x is multiplied by 2, divide both sides by 2 to undo the multiplication.

Check $5x - 2 = 3x + 4$

$5(3) - 2$	$3(3) + 4$
$15 - 2$	$9 + 4$
13	$13 \checkmark$

To check your solution, substitute 3 for x in the original equation.

Helpful Hint

Equations are often easier to solve when the variable has a positive coefficient. Keep this in mind when deciding on which side to “collect” variable terms.



Solve each equation. Check your answer.

1a. $4b + 2 = 3b$

1b. $0.5 + 0.3y = 0.7y - 0.3$

To solve more complicated equations, you may need to first simplify by using the Distributive Property or combining like terms.

EXAMPLE 2 Simplifying Each Side Before Solving Equations

Solve each equation.

A $2(y + 6) = 3y$

$$\begin{aligned} 2(y + 6) &= 3y \\ 2(y) + 2(6) &= 3y \\ 2y + 12 &= 3y \\ \underline{-2y} \quad \underline{-2y} & \\ 12 &= y \end{aligned}$$

Distribute 2 to the expression in parentheses.

To collect the variable terms on one side, subtract $2y$ from both sides.

Check $2(y + 6) = 3y$

$2(12 + 6)$	$3(12)$
$2(18)$	36
36	$36 \checkmark$

To check your solution, substitute 12 for y in the original equation.

B $3 - 5b + 2b = -2 - 2(1 - b)$

$$\begin{aligned} 3 - 5b + 2b &= -2 - 2(1 - b) \\ 3 - 5b + 2b &= -2 - 2(1) - 2(-b) \\ 3 - 5b + 2b &= -2 - 2 + 2b \\ 3 - 3b &= -4 + 2b \\ \underline{+3b} \quad \underline{+3b} & \\ 3 &= -4 + 5b \\ \underline{+4} \quad \underline{+4} & \\ 7 &= 5b \\ \frac{7}{5} &= \frac{5b}{5} \\ 1.4 &= b \end{aligned}$$

Distribute -2 to the expression in parentheses.

Combine like terms.

Add $3b$ to both sides.

Since -4 is added to $5b$, add 4 to both sides.

Since b is multiplied by 5, divide both sides by 5.



Solve each equation. Check your answer.

2a. $\frac{1}{2}(b + 6) = \frac{3}{2}b - 1$

2b. $3x + 15 - 9 = 2(x + 2)$

An **identity** is an equation that is true for all values of the variable. An equation that is an identity has infinitely many solutions. A **contradiction** is an equation that is not true for any value of the variable. It has no solutions.



Identities and Contradictions

WORDS	NUMBERS	ALGEBRA
<p>Identity</p> <p>When solving an equation, if you get an equation that is always true, the original equation is an identity, and it has infinitely many solutions.</p>	$\begin{aligned} 2 + 1 &= 2 + 1 \\ 3 &= 3 \checkmark \end{aligned}$	$\begin{aligned} 2 + x &= 2 + x \\ \underline{-x} \quad \underline{-x} & \\ 2 &= 2 \checkmark \end{aligned}$
<p>Contradiction</p> <p>When solving an equation, if you get a false equation, the original equation is a contradiction, and it has no solutions.</p>	$\begin{aligned} 1 &= 1 + 2 \\ 1 &= 3 \times \end{aligned}$	$\begin{aligned} x &= x + 3 \\ \underline{-x} \quad \underline{-x} & \\ 0 &= 3 \times \end{aligned}$

EXAMPLE 3**Infinitely Many Solutions or No Solutions**

Solve each equation.

A $x + 4 - 6x = 6 - 5x - 2$

$x + 4 - 6x = 6 - 5x - 2$

$4 - 5x = 4 - 5x$

$\underline{+ 5x} \quad \underline{+ 5x}$

$4 = 4 \checkmark$

*Identify like terms.**Combine like terms on the left and the right.**Add 5x to both sides.**True statement*The equation $x + 4 - 6x = 6 - 5x - 2$ is an identity. All values of x will make the equation true. All real numbers are solutions.

B $-8x + 6 + 9x = -17 + x$

$-8x + 6 + 9x = -17 + x$

$x + 6 = -17 + x$

$\underline{-x} \quad \underline{-x}$

$6 = -17 \times$

*Identify like terms.**Combine like terms.**Subtract x from both sides.**False statement*The equation $-8x + 6 + 9x = -17 + x$ is a contradiction. There is no value of x that will make the equation true. There are no solutions.**Writing Math**

The solution set for Example 3B is an empty set—it contains no elements. The empty set can be written as Φ (the Greek letter phi) or $\{\}$.



Solve each equation.

3a. $4y + 7 - y = 10 + 3y$

3b. $2c + 7 + c = -14 + 3c + 21$

EXAMPLE 4**Consumer Application**

The long-distance rates of two phone companies are shown in the table. How long is a call that costs the same amount no matter which company is used? What is the cost of that call?

Phone Company	Charges
Company A	36¢ plus 3¢ per minute
Company B	6¢ per minute

Let m represent minutes, and write expressions for each company's cost.

When is 36¢ plus 3¢ per minute the same as 6¢ per minute?

$36 + 3m = 6m$

$36 + 3m = 6m$

$\underline{- 3m} \quad \underline{- 3m}$
 $36 = 3m$

$\underline{3} \quad \underline{3}$
 $12 = m$

*To collect the variable terms on one side, subtract 3m from both sides.**Since m is multiplied by 3, divide both sides by 3 to undo the multiplication.*The charges will be the same for a 12-minute call using either phone service. To find the cost of this call, evaluate either expression for $m = 12$:

$36 + 3m = 36 + 3(12) = 36 + 36 = 72 \quad 6m = 6(12) = 72$

The cost of a 12-minute call through either company is 72¢.



4. Four times Greg's age, decreased by 3 is equal to 3 times Greg's age, increased by 7. How old is Greg?

THINK AND DISCUSS

1. Tell which of the following is an identity. Explain your answer.

a. $4(a + 3) - 6 = 3(a + 3) - 6$ b. $8.3x - 9 + 0.7x = 2 + 9x - 11$

2. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write an example of an equation that has the indicated number of solutions.

An equation with variables on both sides can have...

one solution:

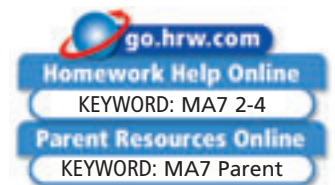
many solutions:

no solution:



2-4

Exercises



GUIDED PRACTICE

1. **Vocabulary** An equation that has no solution is called a(n) _____. (*identity* or *contradiction*)

SEE EXAMPLE 1

p. 100

Solve each equation. Check your answer.

2. $2c - 5 = c + 4$

3. $8r + 4 = 10 + 2r$

4. $2x - 1 = x + 11$

5. $28 - 0.3y = 0.7y - 12$

SEE EXAMPLE 2

p. 101

6. $-2(x + 3) = 4x - 3$

7. $3c - 4c + 1 = 5c + 2 + 3$

8. $5 + 3(q - 4) = 2(q + 1)$

9. $5 - (t + 3) = -1 + 2(t - 3)$

SEE EXAMPLE 3

p. 102

10. $7x - 4 = -2x + 1 + 9x - 5$

11. $8x + 6 - 9x = 2 - x - 15$

12. $6y = 8 - 9 + 6y$

13. $6 - 2x - 1 = 4x + 8 - 6x - 3$

SEE EXAMPLE 4

p. 102

14. **Consumer Economics** A house-painting company charges \$376 plus \$12 per hour. Another painting company charges \$280 plus \$15 per hour.

a. How long is a job for which both companies will charge the same amount?

b. What will that cost be?

PRACTICE AND PROBLEM SOLVING

Solve each equation. Check your answer.

15. $7a - 17 = 4a + 1$

16. $2b - 5 = 8b + 1$

17. $4x - 2 = 3x + 4$

18. $2x - 5 = 4x - 1$

19. $8x - 2 = 3x + 12.25$

20. $5x + 2 = 3x$

21. $3c - 5 = 2c + 5$

22. $-17 - 2x = 6 - x$

23. $3(t - 1) = 9 + t$

24. $5 - x - 2 = 3 + 4x + 5$

25. $2(x + 4) = 3(x - 2)$

26. $3m - 10 = 2(4m - 5)$

27. $5 - (n - 4) = 3(n + 2)$

28. $6(x + 7) - 20 = 6x$

29. $8(x + 1) = 4x - 8$

30. $x - 4 - 3x = -2x - 3 - 1$

31. $-2(x + 2) = -2x + 1$

32. $2(x + 4) - 5 = 2x + 3$

Independent Practice

For Exercises	See Example
15–22	1
23–29	2
30–32	3
33	4

Extra Practice

Skills Practice p. S6

Application Practice p. S29

33. **Sports** Justin and Tyson are beginning an exercise program to train for football season. Justin weighs 150 lb and hopes to gain 2 lb per week. Tyson weighs 195 lb and hopes to lose 1 lb per week.

- If the plan works, in how many weeks will the boys weigh the same amount?
- What will that weight be?

Write an equation to represent each relationship. Then solve the equation.

34. Three times the sum of a number and 4 is the same as 18 more than the number.
35. A number decreased by 30 is the same as 14 minus 3 times the number.
36. Two less than 2 times a number is the same as the number plus 64.

Solve each equation. Check your answer.

37. $2x - 2 = 4x + 6$

38. $3x + 5 = 2x + 2$

39. $4x + 3 = 5x - 4$

40. $-\frac{2}{5}p + 2 = \frac{1}{5}p + 11$

41. $5x + 24 = 2x + 15$

42. $5x - 10 = 14 - 3x$

43. $12 - 6x = 10 - 5x$

44. $5x - 7 = -6x - 29$

45. $1.8x + 2.8 = 2.5x + 2.1$

46. $2.6x + 18 = 2.4x + 22$

47. $1 - 3x = 2x + 8$

48. $\frac{1}{2}(8 - 6h) = h$

49. $3(x + 1) = 2x + 7$

50. $9x - 8 + 4x = 7x + 16$

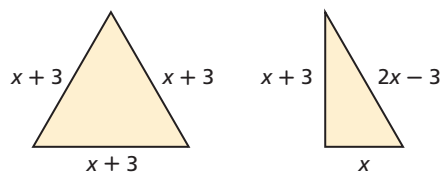
51. $3(2x - 1) + 5 = 6(x + 1)$

52. **Travel** Rapid Rental Car company charges a \$40 rental fee, \$15 for gas, and \$0.25 per mile driven. For the same car, Capital Cars charges \$45 for rental and gas and \$0.35 per mile.

- Find the number of miles for which the companies' charges will be the same. Then find that charge. Show that your answers are reasonable.
- The Barre family estimates that they will drive about 95 miles during their vacation to Hershey, Pennsylvania. Which company should they rent their car from? Explain.
- What if...?** The Barres have extended their vacation and now estimate that they will drive about 120 miles. Should they still rent from the same company as in part b? Why or why not?
- Give a general rule for deciding which company to rent from.



53. **Geometry** The triangles shown have the same perimeter. What is the value of x ?



**MULTI-STEP
TEST PREP**



54. This problem will prepare you for the Multi-Step Test Prep on page 112.
- A fire currently covers 420 acres and continues to spread at a rate of 60 acres per day. How many total acres will be covered in the next 2 days? Show that your answer is reasonable.
 - Write an expression for the total area covered by the fire in d days.
 - The firefighters estimate that they can put out the fire at a rate of 80 acres per day. Write an expression for the total area that the firefighters can put out in d days.
 - Set the expressions in parts b and c equal. Solve for d . What does d represent?



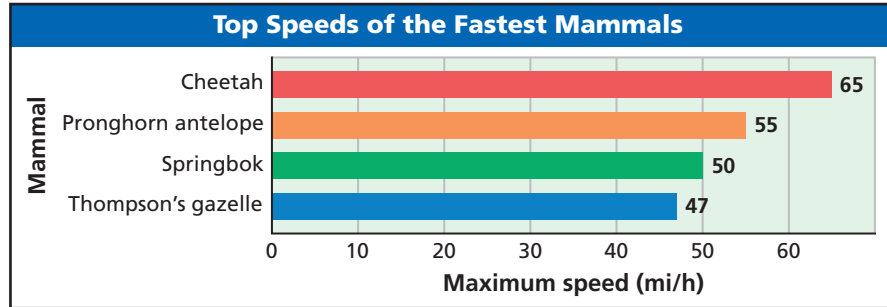
Biology



A cheetah's body is well designed for fast running. Its tail acts like a boat's rudder to help it make sharp turns. Its spine acts like a spring to propel it forward.

Source:
www.cheetahspot.com

55. **Critical Thinking** Write an equation with variables on both sides that has no solution.
56. **Biology** The graph shows the maximum recorded speeds of the four fastest mammals.



Source: *The Top 10 of Everything*

- Write an expression for the distance in miles that a Thompson's gazelle can run at top speed in x hours.
 - Write an expression for the distance in miles that a cheetah can run at top speed in x hours.
 - A cheetah and a Thompson's gazelle are running at their top speeds. The cheetah is one mile behind the gazelle. Write an expression for the distance the cheetah must run to catch up with the gazelle.
 - Write and solve an equation that represents how long the cheetah will have to run at top speed to catch up with the gazelle.
 - A cheetah can maintain its top speed for only 300 yards. Will the cheetah be able to catch the gazelle? Explain.
57. **Write About It** Write a series of steps that you can use to solve any equation with variables on both sides.



58. Lindsey's monthly magazine subscription costs \$1.25 per issue. Kenzie's monthly subscription costs \$1.50 per issue, but she received her first 2 issues free. Which equation can be used to find the number of months after which the girls will have paid the same amount?
- (A) $1.25m = 1.50m - 2$ (C) $1.25m = 1.50(m - 2)$
 (B) $1.25m = 1.50m - 2m$ (D) $1.25m = 3m - 1.50$
59. What is the numerical solution of the equation *7 times a number equals 3 less than 5 times that number*?
- (F) -1.5 (G) 0.25 (H) $\frac{2}{3}$ (J) 4
60. Three packs of markers cost \$9.00 less than 5 packs of markers. Which equation best represents this situation?
- (A) $5x + 9 = 3x$ (B) $3x + 9 = 5x$ (C) $3x - 9 = 5x$ (D) $9 - 3x = 5x$
61. Nicole has \$120. If she saves \$20 per week, in how many days will she have \$500?
- (F) 19 (G) 25 (H) 133 (J) 175
62. **Gridded Response** Solve $-2(x - 1) + 5x = 2(2x - 1)$.

CHALLENGE AND EXTEND

Solve each equation.

63. $4x + 2[4 - 2(x + 2)] = 2x - 4$

64. $\frac{x+5}{2} + \frac{x-1}{2} = \frac{x-1}{3}$

65. $\frac{2}{3}w - \frac{1}{4} = \frac{2}{3}\left(w - \frac{1}{4}\right)$

66. $-5 - 7 - 3f = -f - 2(f + 6)$

67. $\frac{2}{3}x + \frac{1}{2} = \frac{3}{5}x - \frac{5}{6}$

68. $x - \frac{1}{4} = \frac{x}{3} + 7\frac{3}{4}$

69. Find three consecutive integers such that twice the greatest integer is 2 less than 3 times the least integer.

70. Find three consecutive integers such that twice the least integer is 12 more than the greatest integer.

71. Rob had twice as much money as Sam. Then Sam gave Rob 1 quarter, 2 nickels, and 3 pennies. Rob then gave Sam 8 dimes. If they now have the same amount of money, how much money did Rob originally have? Check your answer.

SPIRAL REVIEW

Write an expression for the perimeter of each figure. (Lesson 1-1)

72. square with side x cm

73. equilateral triangle with side y cm

Multiply or divide. (Lesson 1-3)

74. $6.1 \div 0$

75. $3(-21)$

76. $0 \div \frac{7}{8}$

77. $\frac{2}{5} \div \frac{1}{10}$

78. $5 \div (-5)$

79. $\frac{-16}{-8}$

80. $-1000 \div (-0.001)$

81. $500(-0.25)$

Solve each equation. (Lesson 2-3)

82. $4x - 44 = 8$

83. $2(x - 3) = 24$

84. $-1 = \frac{x}{4} - 3$

85. $2x + 6 = 12$

Career Path



Beth Simmons
Biology major

Q: What math classes did you take in high school?

A: Algebra 1 and 2, Geometry, and Precalculus

Q: What math classes have you taken in college?

A: Two calculus classes and a calculus-based physics class

Q: How do you use math?

A: I use math a lot in physics. Sometimes I would think a calculus topic was totally useless, and then we would use it in physics class! In biology, I use math to understand populations.

Q: What career options are you considering?

A: When I graduate, I could teach, or I could go to graduate school and do more research. I have a lot of options.

go.hrw.com
Career Resources Online
KEYWORD: MA7 Career